

Decay of axial-vector mesons into VP and P γ

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(Received 3 December 2003; revised manuscript received 8 June 2004; published 5 November 2004)

We propose a phenomenological Lagrangian for the decay of the SU(3) nonets of the axial-vector mesons of $J^{PC} = 1^{+-}, 1^{++}$ into a vector meson and a pseudoscalar constructed with tensor fields for the vector and axial-vector mesons. The formulation leads to a good reproduction of the different decay branching ratios and assuming vector meson dominance (VMD) it also leads to good results for the radiative decay of the a_1 into pion and photon, and in agreement with the structure proposed in the chiral tensor formulation of radiative decay of axial-vector mesons. The two SU(3) parameters and the mixing angle of K_{1A} and K_{1B} needed to give the physical $K_1(1270)$ and $K_1(1400)$ resonances are also determined.

DOI: 10.1103/PhysRevD.70.094006

PACS numbers: 13.25.Jx, 14.40.Cs, 14.40.Ev

I. INTRODUCTION

The decay of axial-vector mesons of $J^{PC} = 1^{+-}, 1^{++}$ (nonets associated to the $b_1(1235)$ and $a_1(1260)$ and partners, respectively, (see Table I), into a pseudoscalar (P) and a vector meson (V) has received some attention from the perspective of quark models [1,2] as well as using effective Lagrangians of the strong interaction [3–11]. Much of the work has been done in SU(2) [3,6,8–10] and general structures have been developed. In particular, an interesting formulation develops starting from chiral Lagrangians of pseudoscalar mesons assuming a gauge nature for the vector fields and introducing them through covariant derivatives [4–9]. Two main lines of work have developed using the formalisms of hidden symmetry [6] and Yang Mills [4] and connections can be done between them [7,12]. Work has also been done in the SU(3) sector [4,5,7,11], although it has been applied mostly to the SU(2) sector, particularly to the study of the $a_1 \rightarrow \rho\pi$ decay.

In most of the works the vector field formalism for the vector mesons is used. There is another way to deal with spin-1 particles assigning them an antisymmetric tensor field [13,14]. The main difference between the tensor and vector field formalisms stems from the difference in the vector meson propagator using tensor fields. By introducing local terms and with basic assumptions of vector meson dominance, the two formalisms were shown to be equivalent up to $\mathcal{O}(p^4)$ [15] and with only one vector field (which is not the case studied in the present work). Further work on the equivalence of the two formalisms, up to local terms, for general Lagrangians, based on dual transformations of gauge theories, using path integral techniques, is done in [16]. The same kind of equivalence exploring the baryon sector was seen in [17].

The tensor formalism has proved practical in problems of chiral theory in which the vector mesons are explicitly used in the Lagrangians in order to study properties of pseudoscalar meson interaction [18] and pseudoscalar

form factors [19–21], since the vector mesons are genuine resonances which remain in the large N_c limit [18,22], unlike the light scalar mesons which are dynamically generated by the meson-meson interaction itself [18,23]. In chiral dynamical approaches, like the one of [18], they appear as poles in the driving term of the interaction. The use of the tensor formalism in this case is useful to avoid doublecounting of terms that the lowest order Lagrangian would already contain should one use the vector formalism. On the other hand, if one considers the nonabelian anomaly, the Lagrangian accounting for it, obtained from the gauged Wess-Zumino term [24,25], is naturally written in terms of vector meson fields in the vector representation although, as suggested in [16], it could also be represented in terms of tensor fields. This anomalous Lagrangian accounts for processes which do not conserve intrinsic parity, but this is not the case in the axial-vector decay into vector and pseudoscalar.

The Lagrangian that we use in the present work is also written in terms of tensor fields. However, for the purpose of evaluating the $A \rightarrow VP$ decays, which is the main aim of the present work, using the tensor fields, or the analogous $\partial_\mu V_\nu - \partial_\nu V_\mu$ combination of vector field, is equivalent since the Lagrangians are only used at tree level. We shall see a practical justification for the use of the tensor representation, since the use of this formalism, together with the vector meson dominance hypothesis to couple vector mesons to photons, leads to a gauge invariant amplitude in the present case which agrees with the structure and findings for the radiative decays of the a_1 axial-vector meson in the chiral formalism of [14]. This would not be the case using the straight vector formalism.

TABLE I. Nonets of axial-vector mesons.

J^{PC}	$I = 1$	$I = 0$	$I = 1/2$
1^{+-}	$b_1(1235)$	$h_1(1170), h_1(1380)$	K_{1B}
1^{++}	$a_1(1260)$	$f_1(1285), f_1(1420)$	K_{1A}

The main motivation for the present study is that evidence is coming that the AVP vertices play a relevant role in physical processes like radiative ϕ decay [26] and J/Ψ decays [27,28] through sequential mechanisms involving the exchange of vector mesons. These mechanisms were shown to be important in radiative decays of ω , ρ , ϕ in [26,29,30]. The works of [26–28] came to show that the sequential mechanism involving the exchange of axial-vector mesons also play a relevant role in these reactions. With these problems in mind, it becomes more useful to have at hand a Lagrangian which combines simplicity, wide applicability, and accuracy. At the same time, most of the work done, for instance those derived in a chiral formalism [3–11], pay attention to the $a_1(1260)$ meson, while the $b_1(1235)$ has received much less attention. However, both the a_1 and the b_1 mesons and their SU(3) partners appear on an equal footing in the physical processes which we mentioned before. In order to construct the needed Lagrangian, satisfying the mentioned requirements, we will assume SU(3) symmetry and consider parity and charge conjugation of the particles. One then obtains two structures involving the commutator and anticommutator of the SU(3) matrices associated to the V and P fields. Although SU(3) breaking terms could also be present, the results with SU(3) symmetry prove to be accurate enough within present experimental uncertainties in the data. SU(3) is broken anyway through the use of the physical masses. Our Lagrangian contains only one free parameter for the a_1 (and SU(3) partners) and another one for the b_1 (and SU(3) partners) decays. This is in contrast with other models which already contain three or more parameters for the a_1 decay [3–5,7,9–11]. In spite of its simplicity we shall see that our Lagrangians lead to good results for the partial decay widths.

Another issue in our study, implying a new free parameter, is the mixing angle between the strange members of the nonets, the K_{1A} and K_{1B} , necessary to get the physical $K_1(1270)$ and $K_1(1400)$ resonances. The experimental fact that the $K_1(1270)$ decay into ρK is much larger than into $K^*\pi$, and that the $K_1(1400)$ decay into $K^*\pi$ is much larger than into ρK , is a clear indication that there is a large mixing between the strange members of the SU(3) nonets, because SU(3) would imply K_{1A} and K_{1B} to have a similar amplitude for both decay modes. Nevertheless, there is no consensus about the origin of this mixing angle. In [31], a dynamical origin was speculated through the coupling of the two $|K_1\rangle$ states to their decay channels, suggesting the mixing angle to be $\alpha \sim 45^\circ$ provided the K_{1A} and K_{1B} states are degenerate before mixing. Within quark models, a possible explanation comes from the spin-orbit interaction if $m_q \neq m_{\bar{q}}$. Concerning the determination of the mixing angle, most of the works have been based on the fit to the K_1 decays into ρK and $K^*\pi$ and/or to the ratio $B[\tau \rightarrow \nu_\tau K_1(1270)]/B[\tau \rightarrow \nu_\tau K_1(1400)]$, both using symmetry

arguments to relate the amplitudes or either using quark models. Using K_1 decay data, the fit of [32,33] found an angle of $\sim 45^\circ$, from data as of 1976, and in [34] the solutions 35° , 45° , or 57° were found, but the 45° solution was discarded when considering mass formulas from quark models, which can relate the masses between the $I = 1/2$ members with the mixing angle. In this latter paper also the τ decay was considered to refine the results and the author found that the experimental data preferred the solution of 35° . However, few years later the experimental results on τ decay changed and in the recent work of [35] both the 35° and 57° angles were shown to be compatible with the new data. Using relativized quark models, the authors of [36] found a large uncertain value of $-30^\circ \leq \alpha \leq 50^\circ$ from the τ branching ratio and $45^\circ - 50^\circ$ from the K_1 decays. In [37], using nonrelativistic constituent quark models to obtain relations between masses, the authors found $35^\circ \leq \alpha \leq 57^\circ$. In the work of [38], the $3P_0$ model was used, which is based in the assumption that the strong decays take place through the production of a $q\bar{q}$ pair with vacuum quantum numbers, and from the K_1 decays a mixing angle $\sim 45^\circ$ was obtained. Finally, fitting to charmonium and orthocharmonium decays, in [39], a valid range for the mixing angle between 30° and 60° was obtained. The previous account summarizes the status of the knowledge on this mixing angle. The previous works do not use in the fits the data of other $A \rightarrow VP$ decays apart from the K_1 's, neither use information of the dynamics of the amplitude which can be acquired from a suited Lagrangian. In the present work we retake the issue, in view of the accuracy that our Lagrangians provides, and we apply our AVP Lagrangian to get the maximum obtainable information on the mixing angle from the $A \rightarrow VP$ data, and also the $a_1 \rightarrow \pi\gamma$ decay. We will use not only the K_1 decay data but also the known available experimental information of other $A \rightarrow VP$ decays which, although do not depend on the mixing angle, they depend on the couplings of the Lagrangians and consequently influence the global fit. The present study has also the advantage of considering the dynamics given by our Lagrangians, not only the SU(3) relation between the couplings.

II. THE MODEL

For the Lagrangians accounting for the AVP vertices, where A symbolizes the axial-vector mesons, we propose the following expressions

$$\mathcal{L}_{BVP} = \tilde{D} \langle B_{\mu\nu} \{V^{\mu\nu}, P\} \rangle \quad (1)$$

$$\mathcal{L}_{AVP} = i\tilde{F} \langle A_{\mu\nu} [V^{\mu\nu}, P] \rangle$$

where $\langle \rangle$ means SU(3) trace and the i factor in front of the $\tilde{F}$ is needed in order $\mathcal{L}_{AVP}$ to be hermitian.

In Eq. (1) P is the usual SU(3) matrix containing the pseudoscalar mesons and $B_{\mu\nu}$, $A_{\mu\nu}$, $V_{\mu\nu}$ are SU(3) matrices of the tensor fields associated to the axial-vector mesons of the b_1 and a_1 nonet and the ρ vector meson

nonet, respectively. Assuming the same SU(3) mixture for the singlet and the $I = 0$ of the octet that one has for ϕ and ω , the vector meson and axial-vector meson SU(3) matrices $B_{\mu\nu}$, $A_{\mu\nu}$, $V_{\mu\nu}$ are given by

$$\begin{aligned} B_{\mu\nu} &\equiv \begin{pmatrix} \frac{1}{\sqrt{2}}b_1^0 + \frac{1}{\sqrt{2}}h_1(1170) & b_1^+ & K_{1B}^+ \\ b_1^- & -\frac{1}{\sqrt{2}}b_1^0 + \frac{1}{\sqrt{2}}h_1(1170) & K_{1B}^0 \\ K_{1B}^- & \bar{K}_{1B}^0 & h_1(1380) \end{pmatrix}_{\mu\nu}, \\ A_{\mu\nu} &\equiv \begin{pmatrix} \frac{1}{\sqrt{2}}a_1^0 + \frac{1}{\sqrt{2}}f_1(1285) & a_1^+ & K_{1A}^+ \\ a_1^- & -\frac{1}{\sqrt{2}}a_1^0 + \frac{1}{\sqrt{2}}f_1(1285) & K_{1A}^0 \\ K_{1A}^- & \bar{K}_{1A}^0 & f_1(1420) \end{pmatrix}_{\mu\nu}, \\ V_{\mu\nu} &\equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & \rho^+ & K^{*+} \\ \rho^- & -\frac{1}{\sqrt{2}}\rho^0 + \frac{1}{\sqrt{2}}\omega & K^{*0} \\ K^{*-} & \bar{K}^{*0} & \phi \end{pmatrix}_{\mu\nu} \end{aligned} \quad (2)$$

Similarly, the SU(3) matrix, P , for the pseudoscalar mesons, assuming the standard $\eta - \eta'$ mixing [40], is given by

$$P \equiv \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{3}}\eta + \frac{1}{\sqrt{6}}\eta' & K^0 \\ K^- & \bar{K}^0 & -\frac{1}{\sqrt{3}}\eta + \sqrt{\frac{2}{3}}\eta' \end{pmatrix}. \quad (3)$$

In Eq. (1) the fields $W_{\mu\nu} \equiv A_{\mu\nu}, B_{\mu\nu}, V_{\mu\nu}$ are normalized such that [14]

$$\langle 0 | W_{\mu\nu} | W; P, \epsilon \rangle = \frac{i}{M_W} [P_\mu \epsilon_\nu(W) - P_\nu \epsilon_\mu(W)] \quad (4)$$

Equation (4) is illustrating because it tells us that at tree level we would obtain the same results using the combination $\partial_\mu W_\nu - \partial_\nu W_\mu$ instead of $W_{\mu\nu}$, with W_μ satisfying the Proca equation, i.e., vector formalism for the spin-1 fields.

The structure of the Lagrangians of Eq. (1) is SU(3) invariant and preserves parity, P , and charge conjugation, C .

Should we have used vector fields without derivatives instead of tensor fields in Eq. (1), the structure that would appear would be the minimal one, in the sense of number of derivatives, satisfying the SU(3), parity and charge conjugation symmetries. For the case of the 1^{++} nonet it would be of the form

$$\langle A_\mu [V^\mu, P] \rangle, \quad (5)$$

which was already derived in [6] and applied to the SU(2) sector. This structure appears also naturally in the chiral

formalism of [4,5,7] in addition to other nonminimal terms (see¹ Eq. (3.9) of [4] or (2.59) of [7]).

$$\begin{aligned} &\langle (\partial_\mu V_\nu - \partial_\nu V_\mu) [A^\mu, \partial^\nu \phi] \rangle, \\ &\langle (\partial_\mu A_\nu - \partial_\nu A_\mu) [V^\mu, \partial^\nu \phi] \rangle, \\ &\langle (\partial_\mu A_\nu - \partial_\nu A_\mu) [(\partial^\mu V^\nu - \partial^\nu V^\mu), \phi] \rangle. \end{aligned} \quad (6)$$

A particular combination of the first two terms in Eq. (6) also appears in the chiral formalism of [8]. The last term of Eq. (6) corresponds to our Lagrangian of Eq. (1) for tree level calculations, as we discussed before, given the matrix element of Eq. (4). One can prove that the first and second terms of Eq. (6) can be cast in terms of the third one and the one of Eq. (5) by using part integration. This reduces the freedom in the Lagrangian, at the level of up to two derivatives, to our tensor structure plus the vector one of Eq. (5). Note, however, that the use of derivatives in the present case does not necessarily mean higher orders

¹There are misprints in the last term of those equations: a commutator symbol in [4,7] and an extra ϕ field in [7] are missing [41].

TABLE II. λ_{AVP} for the b_1 , a_1 , $h \equiv h_1(1170)$, $h' \equiv h_1(1380)$, $f \equiv f_1(1285)$ and $f' \equiv f_1(1420)$ decays into VP . The coefficients involved in the decay of b_1 , h and h' have to be multiplied by $\tilde{D}$ and those involved in the decay of a_1 , f and f' by $i\tilde{F}$. The $(-)$ symbol over some K means anti- K when corresponding.

$b_1^\pm \rightarrow K^* K$	$b_1^0 \rightarrow K^{*\pm} K^\mp$	$b_1^0 \rightarrow K^{*0} K^0$
1	$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$
$b_1 \rightarrow \omega \pi$	$b_1 \rightarrow \rho \eta$	$b_1 \rightarrow \rho \eta'$
$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{3}}$
$a_1^\pm \rightarrow \rho^\pm \pi^0$	$a_1^\pm \rightarrow \rho^0 \pi^\pm$	$a_1^0 \rightarrow \rho^\pm \pi^\mp$
$\pm \sqrt{2}$	$\mp \sqrt{2}$	$\mp \sqrt{2}$
$a_1^\pm \rightarrow K^{*\pm} K^{(-)}$	$a_1^\pm \rightarrow K^{*0} K^\pm$	$a_1^0 \rightarrow K^{*\pm} K^\mp$
∓ 1	± 1	$\mp \frac{1}{\sqrt{2}}$
$a_1^0 \rightarrow K^{*0} \bar{K}^0$	$a_1^0 \rightarrow \bar{K}^{*0} K^0$	
$\frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	
$h \rightarrow \rho \pi$	$h \rightarrow \omega \eta$	$h \rightarrow \omega \eta'$
$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	$\frac{\sqrt{2}}{\sqrt{3}}$
$h \rightarrow K^* K$		
$\frac{1}{\sqrt{2}}$		
$h' \rightarrow K^* K$	$h' \rightarrow \phi \eta$	$h' \rightarrow \phi \eta'$
1	$\frac{-2}{\sqrt{3}}$	$2\frac{\sqrt{2}}{\sqrt{3}}$
$f \rightarrow K^{*\pm} K^\mp$	$f \rightarrow K^{*0} \bar{K}^0$	$f \rightarrow \bar{K}^{*0} K^0$
$\mp \frac{1}{\sqrt{2}}$	$\frac{-1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$f' \rightarrow K^{*\pm} K^\mp$	$f' \rightarrow K^{*0} \bar{K}^0$	$f' \rightarrow \bar{K}^{*0} K^0$
± 1	1	-1

in a perturbation expansion in a small momenta, since derivatives on the vector fields can produce masses of the massive vectors. As mentioned in the introduction, we shall use only the structure of Eq. (1), by means of which

a good reproduction of the data is obtained in the SU(3) sector and which naturally leads, via VMD, to a gauge invariant $A \rightarrow V\gamma$ amplitude as we shall see. We shall also discuss what difference one expects for the $A \rightarrow VP$ decay by using the vector term alone of Eq. (5).

The width for the decay $A \rightarrow VP$ or $B \rightarrow VP$ is given by

$$\Gamma_{A \rightarrow VP} = \frac{q}{8\pi M_A^2} \sum |t|^2 \quad (7)$$

where $q = \frac{1}{2M_A} \lambda^{1/2}(M_A^2, M_V^2, M_P^2)$ is the momentum of the final particles in the axial-vector rest frame and the t matrix is given by

$$t_{A \rightarrow VP} = \frac{-2\lambda_{AVP}}{M_A M_V} (p' \cdot p \epsilon' \cdot \epsilon - \epsilon' \cdot p \epsilon \cdot p') \quad (8)$$

where p', ϵ' , and p, ϵ are the momenta and polarization vectors of the axial-vector and the vector mesons, respectively. The λ_{AVP} are the coefficient of the AVP vertex obtained using the Lagrangians of Eq. (1).

The resulting decay width is

$$\Gamma_{A \rightarrow VP} = \frac{|\lambda_{AVP}|^2}{2\pi M_A^2} q \left(1 + \frac{2}{3} \frac{q^2}{M_V^2} \right). \quad (9)$$

The coefficients λ_{AVP} in Eq. (9) for the different reactions are given in Tables II and III. The momentum structure of Eq. (9), with the weight $2/3$ in the q^2/M_V^2 term, is the same as the one obtained in [8]. Later on we shall relate the coefficients λ_{AVP} to the pion decay constant, f , and will see that our expression coincides with the one obtained in [8] for the case $a_1 \rightarrow \rho \pi$.

In the case where there is little phase space for the decay or it takes place due to the width of the particles, we fold the expression for the width with the mass distribution of the particles as

TABLE III. λ_{AVP} for the decay of $K_1(1270)$ into PV . In the coefficients, $c \equiv \cos(\alpha)$ and $s \equiv \sin(\alpha)$. The coefficients for the decays of the $K_1(1400)$ are the same but changing $F \rightarrow -F$ and interchanging $s \leftrightarrow c$.

$K_1^\pm \rightarrow \rho^0 K^\pm$	$K_1^\pm \rightarrow \rho^\pm K^0$	$\bar{K}_1^0 \rightarrow \rho^\mp K^\pm$	$\bar{K}_1^0 \rightarrow \rho^0 \bar{K}^0$
$\pm \frac{1}{\sqrt{2}}(c\tilde{D} - s\tilde{F})$	$\pm(c\tilde{D} - s\tilde{F})$	$\pm(c\tilde{D} - s\tilde{F})$	$(+) - \frac{1}{\sqrt{2}}(c\tilde{D} - s\tilde{F})$
$K_1^\pm \rightarrow K^\pm \omega$	$K_1^0 \rightarrow K^0 \omega$	$\bar{K}_1^0 \rightarrow \bar{K}^0 \omega$	
$\pm \frac{1}{\sqrt{2}}(c\tilde{D} - s\tilde{F})$	$\frac{1}{\sqrt{2}}(c\tilde{D} - s\tilde{F})$	$\frac{-1}{\sqrt{2}}(c\tilde{D} - s\tilde{F})$	
$K_1^\pm \rightarrow K^\pm \phi$	$K_1^0 \rightarrow K^0 \phi$	$\bar{K}_1^0 \rightarrow \bar{K}^0 \phi$	
$\pm(c\tilde{D} + s\tilde{F})$	$c\tilde{D} + s\tilde{F}$	$-(c\tilde{D} + s\tilde{F})$	
$K_1^\pm \rightarrow K^{*\pm} \pi^0$	$K_1^\pm \rightarrow K^{*0} \pi^\pm$	$\bar{K}_1^0 \rightarrow K^{*\pm} \pi^\mp$	$\bar{K}_1^0 \rightarrow K^{*0} \pi^0$
$\pm \frac{1}{\sqrt{2}}(c\tilde{D} + s\tilde{F})$	$\pm(c\tilde{D} + s\tilde{F})$	$\pm(c\tilde{D} + s\tilde{F})$	$(+) - \frac{1}{\sqrt{2}}(c\tilde{D} + s\tilde{F})$
$K_1^\pm \rightarrow K^{*\pm} \eta$	$K_1^0 \rightarrow K^{*0} \eta$	$\bar{K}_1^0 \rightarrow \bar{K}^{*0} \eta$	
$\pm \frac{2}{\sqrt{3}}s\tilde{F}$	$\frac{2}{\sqrt{3}}s\tilde{F}$	$\frac{-2}{\sqrt{3}}s\tilde{F}$	
$K_1^\pm \rightarrow K^{*\pm} \eta'$	$K_1^0 \rightarrow K^{*0} \eta'$	$\bar{K}_1^0 \rightarrow \bar{K}^{*0} \eta'$	
$\pm \sqrt{\frac{3}{2}}(c\tilde{D} - \frac{1}{3}s\tilde{F})$	$\sqrt{\frac{3}{2}}(c\tilde{D} - \frac{1}{3}s\tilde{F})$	$-\sqrt{\frac{3}{2}}(c\tilde{D} - \frac{1}{3}s\tilde{F})$	

$$\Gamma_{A \rightarrow VP} = \frac{1}{\pi^2} \int ds_A ds_V \text{Im} \left\{ \frac{1}{s_A - M_A^2 + iM_A \Gamma_A} \right\} \text{Im} \left\{ \frac{1}{s_V - M_V^2 + iM_V \Gamma_V} \right\} \Gamma_{AVP}(\sqrt{s_A}, \sqrt{s_V}) \Theta(\sqrt{s_A} - \sqrt{s_V} - M_P) \quad (10)$$

where Θ is the step function, $\Gamma_{AVP} = \frac{|\lambda_{AVP}|^2}{2\pi s_A} q \left(1 + \frac{2}{3} \frac{q^2}{s_V} \right)$ with $q = \frac{1}{2\sqrt{s_A}} \lambda^{1/2}(s_A, s_V, M_P^2)$.

On the other hand for the decay of the $K_1(1270)$ and $K_1(1400)$ we have to assume a mixing of the type

$$\begin{aligned} K_1(1270) &= \cos(\alpha) K_{1B} - i \sin(\alpha) K_{1A}, \\ K_1(1400) &= \sin(\alpha) K_{1B} + i \cos(\alpha) K_{1A} \end{aligned} \quad (11)$$

where the prescription is taken to have α comparable to the definition in [32–34, 36, 38], where no explicit i factor in the Lagrangian of Eq. (1) is considered, which we wrote here explicitly in order to have a hermitian Lagrangian.

The coefficients in Eq. (9) for the decay of the $K_1(1270)$ and $K_1(1400)$ into different channels are written in Table III.

III. RESULTS AND DISCUSSION

Considering the values given in the particle data group (PDG) table [42] for the decay widths (see Table IV), we carry out a best fit to these data to obtain the $\tilde{D}$, $\tilde{F}$ and α parameters.

In Table IV we show the decays for which there are data for the branching ratios with their errors. For the K_1 decays, the ratios taken are as in the PDG [42]. In the case when the decay has been seen but no numbers are provided in the PDG, we assume the value $(50 \pm 50)\%$ for the branching ratio, implying that in the fit we put as experimental input that the partial decay width is smaller than the total width. These data will generally weigh little in the fit but including them prevents solutions with partial decay widths for some channels unreasonably

TABLE IV. Reactions considered in the fit to obtain the D , F and α parameters. The data for the K_1 decays are taken from [42]. For the other reactions there are no explicit data but we can infer from there an approximate value with a large uncertainty.

Reaction	BR %	Partial width (MeV)
$K_1(1270) \rightarrow \rho K$	42 ± 6	38 ± 10
$K_1(1270) \rightarrow K^* \pi$	16 ± 5	14 ± 6
$K_1(1270) \rightarrow \omega K$	11 ± 2	10 ± 3
$K_1(1400) \rightarrow K^* \pi$	94 ± 6	164 ± 16
$K_1(1400) \rightarrow \rho K$	3 ± 3	5 ± 5
$K_1(1400) \rightarrow \omega K$	1 ± 1	1.7 ± 1.7
$b_1 \rightarrow \omega \pi$	75 ± 25	110 ± 40
$f' \rightarrow K^* K$	50 ± 25	28 ± 14
$h \rightarrow \rho \pi$	50 ± 50	180 ± 180
$a_1 \rightarrow \rho \pi$	50 ± 50	210 ± 210
$h' \rightarrow K^* K$	50 ± 50	45 ± 45

larger than the total width. In the case where the PDG gives only a partial decay width as “dominant” with no number, ($b_1 \rightarrow \omega \pi$), we have taken for the fit a branching ratio of $(75 \pm 25)\%$ which allows any value from more than half to total. In the case where there are two channels presented as dominant, ($f' \rightarrow K^* K$), we have taken $(50 \pm 25)\%$ branching ratio for each of these channels.

We observe that the $K_1(1270)$ meson decays largely into $K\rho$ with the $K^*\pi$ channel suppressed, while the $K_1(1400)$ decays largely into $K^*\pi$ with the $K\rho$ channel suppressed. This feature is what demands the large mixing between the K_{1A} and K_{1B} . We also see that the a_1 decays into $\rho\pi$, with the $\omega\pi$ decay mode forbidden, while the b_1 decays into $\omega\pi$.

As we can see in Table IV, the decays of $K_1(1270)$ and $K_1(1400)$ have good experimental data. But the theoretical expression for these decays manifests certain symmetries under some transformation of the parameters: First of all the theoretical value for the decay is exactly the same if one interchanges $\tilde{D}$ and $\tilde{F}$ and changes $\alpha \rightarrow \pi/2 - \alpha$. The other symmetry is manifested if one makes the following substitutions: $\tilde{D} \rightarrow -\sin(\alpha) \times \sqrt{\tilde{D}^2 + \tilde{F}^2}$, $\tilde{F} \rightarrow \cos(\alpha) \sqrt{\tilde{D}^2 + \tilde{F}^2}$ and $\alpha \rightarrow \arctan\left(\frac{-\tilde{D}}{\tilde{F}}\right)$.

These two symmetries² lead to four different set of parameters as mathematically equivalent solutions if one only considers the $K_1(1270)$ and $K_1(1400)$ decays, which are shown in Table V.

These solutions are similar to those found in [34]. These symmetries in the solutions of the fit can be broken if one introduces other decays which do not depend on α . The problem is that there are very few of these data in the PDG, only from $a_1 \rightarrow K^* K$ one could infer a reasonable fair value for the branching ratio to be used in the fit. But this reaction is told in the PDG to be controversial and then we do not use it in the fit.

We next include also in the fit the last five channels of Table IV, corresponding to “seen” or “dominant” in the PDG, with prescription for the branching ratios and errors explained above. The results for the parameters obtained from the fit are shown in Table VI.

We can see that the results are very similar to those found in Table V because the uncertainties of the new data included are very large and weigh very little in the fit. The main novelty of the new fit is that the solution of $\alpha = 42^\circ$ in Table V disappears and, as seen in Table VI, it seems to prefer the solutions (1) and (2) to the one of $\alpha = 47^\circ$. This

²Apart from these symmetries, typical only for the K_1 decays, there is another symmetry in all the $A \rightarrow PV$ decays in the global sign of $\tilde{D}$ and $\tilde{F}$ but with a fixed value for $\tilde{D}\tilde{F}$ if the mixing angle α is restricted to be between 0° and 90° .

TABLE V. Results of the fit to the data including only the $K_1(1270)$ and $K_1(1400)$ decays. (dof is the number of degrees of freedom dof = (#exp. data) - (#parameters)).

$\tilde{D}$ (MeV)	$\tilde{F}$ (MeV)	α (degrees)	χ^2/dof
-1250 ± 80	1400 ± 100	62 ± 3	1.394
-1650 ± 100	880 ± 90	42 ± 3	1.394
-1400 ± 100	1250 ± 80	28 ± 3	1.394
-880 ± 80	1650 ± 100	48 ± 3	1.394

conclusion would be the same one obtained in [34] for different reasons, since the extra decays evaluated here were not considered in [34]. In any case we should note that the precise value of the χ^2 function in Table VI is tied to the way the unknown data have been entered in the fit (see Table IV) and that, in any case, a fit with a χ^2/dof close to 1 is an acceptable solution.

We now comment on the differences of using the Lagrangian of Eq. (5) instead of the one of Eq. (1) used so far. In the vector formalism, see Eq. (5), the amplitude and width are given by

$$t = -\lambda_{AVP}\epsilon' \cdot \epsilon; \quad \Gamma_{A \rightarrow V\gamma} = \frac{|\lambda_{AVP}|^2}{8\pi M_A^2} q \left(1 + \frac{1}{3} \frac{q^2}{M_V^2} \right), \quad (12)$$

where now $\tilde{D}$ and $\tilde{F}$ would have a different normalization. The difference of Eq. (12) with Eq. (9) is a factor 2 in the q^2 term in the bracket. However, this term is reasonable small in all the decay channels and thus the numerical differences in the decay rates between the two formalisms are very small. Later on we shall nevertheless show that using the tensor formalism leads naturally to a gauge invariant amplitude for the radiative decay of the a_1 resonance, which is not the case if one uses the Lagrangian of Eq. (5).

Next we pass to study the radiative decay of the a_1 and b_1 into $\pi\gamma$. We assume vector meson dominance and hence the mechanism for the decay is represented by the Feynman diagram of Fig. 1.

The case of the a_1 decay proceeds with the exchange of a ρ meson while the decay of the b_1 requires the exchange of the ω meson. In addition to the Lagrangians which we have, we need the vector meson-photon coupling which is given in [14] in the tensor formalism by

TABLE VI. Results of the fit including the data of Table IV.

	$\tilde{D}$ (MeV)	$\tilde{F}$ (MeV)	α (degrees)	χ^2/dof
(1)	-1240 ± 80	1380 ± 100	62 ± 3	0.687
(2)	-1330 ± 90	1250 ± 80	29 ± 3	0.746
(3)	-980 ± 100	1520 ± 100	47 ± 4	1.207

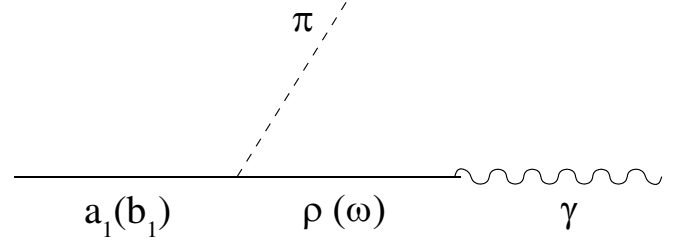


FIG. 1. Diagram for the a_1 and b_1 decay into $\pi\gamma$ through vector meson dominance.

$$\mathcal{L}_{V\gamma} = -e \frac{F_V}{2} \lambda_{V\gamma} V_{\mu\nu}^0 (\partial^\mu A^\nu - \partial^\nu A^\mu) \quad (13)$$

with $\lambda_{V\gamma} = 1, \frac{1}{3}, -\frac{\sqrt{2}}{3}$ for $V^0 = \rho^0, \omega, \phi$ respectively.

Furthermore one also needs the vector meson propagator in the tensor formalism which is also given in [14] as

$$\begin{aligned} \langle 0 | T \{ W_{\mu\nu} W_{\rho\sigma} \} | 0 \rangle &= i \mathcal{D}_{\mu\nu\rho\sigma} \\ &= i \frac{M_W^{-2}}{M_W^2 - P^2 - i\epsilon} [g_{\mu\rho} g_{\nu\sigma} (M_W^2 - P^2) \\ &\quad + g_{\mu\rho} P_\nu P_\sigma - g_{\mu\sigma} P_\nu P_\rho - (\mu \leftrightarrow \nu)] \end{aligned} \quad (14)$$

In view of this, the decay width is given by Eq. (7), where t is now given by

$$t = \frac{2\lambda_{AVP}\lambda_{V\gamma}F_V e}{M_A M_V^2} (p' \cdot p \epsilon' \cdot \epsilon - \epsilon' \cdot p \epsilon \cdot p') \quad (15)$$

which exhibits explicitly gauge invariance. Should we have used the vector couplings of Eq. (5) for the AVP plus the vector formalism for the vector-photon coupling of [40], we would have obtained only the term $\epsilon' \cdot \epsilon$ which does not fulfill gauge invariance and is unacceptable to represent the process, something that was already pointed out in [43]. On the other hand, should we have use Eqs. (1) and (13), replacing $W_{\mu\nu}$ by $\partial_\mu W_\nu - \partial_\nu W_\mu$ and using the standard propagator for $\partial_\mu W_\nu - \partial_\nu W_\mu$, (the same as Eq. (14) except for the contact terms proportional to $(M_W^2 - P^2)$), we would have got zero, indicating that one would have to add contact terms in the vector formalism to make it equivalent to the tensor one, in the line of the claims made in other works trying to show the equivalence between the vector and tensor formalisms [12,15–17].

Using the couplings obtained before for the AVP vertices plus those of the $V\gamma$ vertex, we find for the radiative decay width of the a_1 resonance of 380 ± 50 KeV, 320 ± 40 KeV or 470 ± 60 KeV for the solutions (1), (2) and (3) of Table VI respectively, in reasonable good agreement with the experimental value 640 ± 246 KeV. This is not the case of the b_1 decay for which we get a width 32 ± 4 KeV, 36 ± 5 KeV or 19 ± 4 KeV for the solutions (1),

(2) and (3), which is too small compared with experiment, 230 ± 60 KeV. The reason for this drastic reduction from the a_1 to the b_1 is the factor $1/3$ of the $\omega\gamma$ coupling with respect to the $\rho\gamma$ coupling. Since for this reason the vector meson dominance term is so much reduced with respect to the case of the a_1 decay, it is not surprising that other mechanisms can also account for this decay channel. However, the $b_1 \rightarrow \pi\gamma$ decay width is still much smaller than that of the a_1 , around a factor 3. Such possible mechanisms would thus be smaller than the vector meson dominance mechanisms for the case of the a_1 decay, in which case the decay width provided by the vector meson dominance mechanism should be relatively accurate as it is the case, accounting for more than $2/3$ of the total $a_1 \rightarrow \pi\gamma$ decay width.

It is interesting to establish comparison of the result obtained here for the radiative decay width of the a_1 resonance with those of [14]. This comparison can be carried out at the analytical level by recalling the structure of the Lagrangian in [14]

$$\mathcal{L} = \frac{F_V}{2\sqrt{2}} \langle A_{\mu\nu} f^{\mu\nu} \rangle \quad (16)$$

where $f^{\mu\nu}$ is defined in [14] and provides, in our case, the pseudoscalars and photon fields. From the Lagrangian of Eq. (16) one can derive the amplitude for the $a_1^+ \rightarrow \pi^+ \gamma$ decay obtaining

$$t = -\frac{iF_A e}{fM_A} (p' \cdot p \epsilon' \cdot \epsilon - \epsilon' \cdot p \epsilon \cdot p') \quad (17)$$

where $f = 92.4$ MeV and e is taken positive. This latter amplitude can be compared to the one in Eq. (15) using that for the $a_1^+ \rightarrow \pi^+ \gamma$ decay $\lambda_{V\gamma} = 1$ and $\lambda_{AVP} = -\sqrt{2}iF$ obtaining that

$$\tilde{F} = \frac{1}{2\sqrt{2}} \frac{F_A}{F_V} \frac{M_V^2}{f} \quad (18)$$

It is interesting to recall that the F_A parameter is related, through the Weinberg sum rule [44], to the F_V and f parameters by $F_A = \sqrt{F_V^2 - f^2}$, and using values of F_V and f from [14], then $F_A \simeq 123$ MeV. Using this value in Eq. (18) one obtains $\tilde{F} \simeq 1800$ MeV, to be compared with the values obtained in Table VI. It is also interesting to recall that F_A is related to the L_{10} coefficient of the second order meson chiral Lagrangian [45] through $L_{10}^{V+A} = -F_V^2/4M_V^2 + F_A^2/4M_A^2$. With the value obtained in our fit for $\tilde{F}$, and consequently for F_A , the same qualitative agreement obtained in [14] with the empirical value of L_{10} holds also here. This allows to relate the parameter $\tilde{F}$ of our effective Lagrangian (Eq. (1)) with the L_{10} parameter of the chiral Lagrangians.

Apart from the comparison of the coefficients at the numerical level, it is also illustrative to compare the analytical expression of the coefficient of Eq. (18) with the corresponding one used in [8]. By using the vector

meson dominance values of [14], $F_V = \sqrt{2}f$, $F_A = f$, $M_A = \sqrt{2}M_V$, we obtain $\tilde{F} = M_V^2/4f$, and considering the $a_1^+ \rightarrow \rho^+ \pi^0 + \rho^0 \pi^+$ and the coefficients of λ_{AVP} in Table II, we obtain, for the coefficient of Eq. (9),

$$\frac{|\lambda_{a_1^+ \rho^+ \pi^0}|^2 + |\lambda_{a_1^+ \rho^0 \pi^+}|^2}{2\pi M_A^2} = \frac{M_\rho^2}{16\pi f^2} \quad (19)$$

which coincides with Eq. (2.22) of [8].

We can also perform another study including the experimental $a_1 \rightarrow \pi\gamma$ decay width into the fit, in which case we obtain:

With this final set of parameters we can predict the widths of all the $A \rightarrow VP$ decay widths not included in the particle data table. The results for the three possible solutions of Table VII are shown in Table VIII. We make predictions for all possible decays, many of them yet unobserved. The errors quoted are statistical, but large uncertainties should be assumed in cases where the phase space is only allowed due to the tails of the resonance mass distributions through Eq. (10). It is however instructive to see that all widths predicted are well within the values of the total width of the decaying particle, which we have also written in the table for comparison.

Recently, a paper dealing with $\tau \rightarrow \pi\pi\pi\nu_\tau$ has appeared [46] in which five different structures to account for the AVP coupling in the tensor formalism have been derived. Although the structures derived there are formally different to the one we propose, it is easy to see that at tree level the terms O^4 and O^5 of Eq. (12) of [46] give an identical structure for the amplitude to that of our Lagrangian and so would do a linear combination of O^2 and O^3 . The O^1 , however, breaks explicitly SU(3) symmetry and hence has no room in our SU(3) symmetric approach. In practical terms, the results of the present work indicate that should one use the formalism of [46] at tree level to deal with $A \rightarrow VP$ decay, one would obtain very good agreement with the data by taking, for instance, the O^4 term alone. This, of course, affects only the octet of the a_1 , not the b_1 which we have also studied here.

IV. SUMMARY

We have addressed the decay of an axial-vector into a vector and a pseudoscalar meson looking for a Lagrangian which can reproduce all existing data while making predictions for all yet unobserved allowed chan-

TABLE VII. Results of the fit including the data of Table IV and the $a_1 \rightarrow \pi\gamma$ decay.

	$\tilde{D}$ (MeV)	$\tilde{F}$ (MeV)	α (degrees)	χ^2/dof
(1)	-1230 ± 80	1380 ± 90	62 ± 3	0.743
(2)	-1320 ± 90	1270 ± 80	29 ± 3	0.859
(3)	-960 ± 90	1540 ± 100	47 ± 4	1.129

TABLE VIII. Partial widths obtained for all the $A \rightarrow VP$ decays with the three different possible solutions of the $\tilde{D}$, $\tilde{F}$ and α parameters. The theoretical errors shown are only due to the uncertainties in these parameters.

Reaction	$\Gamma_{\text{tot}}^{\text{exp}}(\text{MeV})$	$\Gamma_i^{\text{exp}}(\text{MeV})$	$\Gamma_i(\text{MeV})$ (1)	$\Gamma_i(\text{MeV})$ (2)	$\Gamma_i(\text{MeV})$ (3)
$a_1 \rightarrow \pi\gamma$	425 ± 175	0.64 ± 0.25	0.37 ± 0.05	0.31 ± 0.04	0.46 ± 0.06
$K_1(1270) \rightarrow \rho K$	90 ± 20	38 ± 10	48 ± 5	46 ± 5	47 ± 5
$K_1(1270) \rightarrow K^* \pi$		14 ± 6	10 ± 4	8 ± 4	6 ± 4
$K_1(1270) \rightarrow \omega K$		10 ± 3	12.8 ± 1.3	12.4 ± 1.3	12.6 ± 1.4
$K_1(1400) \rightarrow K^* \pi$	174 ± 13	164 ± 16	143 ± 14	145 ± 14	146 ± 17
$K_1(1400) \rightarrow \rho K$		5 ± 5	6 ± 4	7 ± 4	4 ± 4
$K_1(1400) \rightarrow \omega K$		1.7 ± 1.7	2.1 ± 1.2	2.4 ± 1.3	1.3 ± 1.2
$b_1 \rightarrow \omega \pi$	142 ± 9	110 ± 40	114 ± 14	130 ± 18	69 ± 14
$f' \rightarrow K^* K$	55.5 ± 2.9	28 ± 14	42 ± 5	34 ± 4	51 ± 6
$h \rightarrow \rho \pi$	360 ± 40	180 ± 180	290 ± 40	330 ± 50	180 ± 30
$a_1 \rightarrow \rho \pi$		210 ± 210	260 ± 30	220 ± 30	320 ± 40
$h' \rightarrow K^* K$	91 ± 30	45 ± 45	42 ± 5	48 ± 7	25 ± 5
$a_1 \rightarrow K^* K$			31 ± 4	26 ± 3	39 ± 5
$b_1 \rightarrow K^* K$			9.1 ± 1.1	10.5 ± 1.4	5.6 ± 1.1
$b_1 \rightarrow \rho \eta$			16 ± 2	18 ± 3	9.8 ± 1.9
$b_1 \rightarrow \rho \eta'$			0.81 ± 0.10	0.93 ± 0.13	0.49 ± 0.10
$h \rightarrow \omega \eta$			17 ± 2	19 ± 3	10 ± 2
$h \rightarrow \omega \eta'$			2.5 ± 0.3	2.9 ± 0.4	1.6 ± 0.3
$h \rightarrow K^* K$			17.7 ± 2	20 ± 3	11 ± 2
$h' \rightarrow \phi \eta$			2.2 ± 0.3	2.6 ± 0.4	1.4 ± 0.3
$h' \rightarrow \phi \eta'$			0.43 ± 0.05	0.49 ± 0.07	0.26 ± 0.05
$f \rightarrow K^* K$	24.0 ± 1.2		3.8 ± 0.5	3.2 ± 0.4	4.8 ± 0.6

nels involving all the particles of the SU(3) nonets. We found that a basic Lagrangian involving commutator and anticommutator of the fields, using the tensor representation for the vector and axial-vector mesons, was rather accurate and, at the same time, simple enough to be used in intermediate steps of more complicated hadronic processes.

We have also shown that the combination of our AVP Lagrangian with Vector Meson Dominance leads to an amplitude for the radiative decay of the a_1 into $\pi\gamma$, which formally agrees with the one obtained in the chiral formulation of vector meson and axial couplings, relating the $\tilde{F}$ parameter of the AVP coupling to the F_V parameter of the $V\gamma$ coupling in VMD and to the coefficients of the meson chiral Lagrangians. The tensor formalism produces small numerical differences in the predictions for the different decays of the axial-vector mesons with respect to the vector formalism without derivatives, yet it leads naturally to a gauge invariant amplitude for the radiative decay of the a_1 resonance while this vector formalism leads to a noninvariant one. From the studied strong decays we found three acceptable solutions for the parameters and the mixing angle of the strange axial-vectors, with two of them with angles around 30° and 60° , slightly favored with respect the solution around 45° . This is the maximum information that can be obtained from all the $A \rightarrow VP$ present decay data. The present determination of the mixing angle has the advantage

from previous works of using all the $A \rightarrow VP$ available decay data, not only K_1 decays, and of considering the dynamics given by a suitable Lagrangian. Regarding the prediction for unobserved channels, it is interesting to observe that all the predicted decay rates are well within the boundaries of the total decay widths. Since the predictions of the three different solutions for some channels are quite different, the measurement of some of them would be most welcome in order to find out the actual mixing and the value of the coupling parameters.

The simple form derived for the Lagrangian has made easier the implementation of mechanisms involving exchange of axial-vector mesons which contribute in processes of radiative decays of ϕ and J/Ψ which had not been discussed until recently. With tests of hadronic models and particularly chiral dynamics been conducted in physical processes occurring at higher energies, an increasing attention will have to be paid to the role of axial-vector mesons. The work in the present paper makes this task easier.

ACKNOWLEDGMENTS

We would like to acknowledge useful discussions with J. Portolés. Two of us, J. E. P. and L. R., acknowledge support from the Ministerio de Educación y Ciencia. This work is partly supported by DGICYT Contract No. BFM2003-00856, and the E.U. EURIDICE network Contract No. HPRN-CT-2002-00311.

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